EE 2240

Problem #13

For the system described by $\ddot{x}+12\dot{x}+32x=0$:

a. Determine the characteristic equation.

$$r^2+12r+32=0$$

b. Determine the natural frequency, ω_n.

$$\omega_n=\sqrt{32}=4\sqrt{2}\approx 5.66$$

c. Determine the damping ratio, ζ.

$$2\zeta\omega_n=12 \Rightarrow \zeta=\frac{12}{2\omega_n}=\frac{6}{\sqrt{32}}=\frac{3}{2\sqrt{2}}\approx 1.66$$

d. Determine the numerical values of the two roots of the characteristic equation.

$$r^2+12r+32=(r+4)(r+8) \Rightarrow r=-4,-8$$

e. Classify the system as *overdamped*, *critically damped*, *underdamped*, or *undamped*.

The system is overdamped ($\zeta>1$)

f. Assuming $x(0)=8$ and $\dot{x}(0)=-44$, determine the solution of the given equation.

$$x(t)=K_1e^{-4t}+K_2e^{-8t}$$

$$\dot{x}(t)=-4K_1e^{-4t}-8K_2e^{-8t}$$

$$\begin{cases} x(0)=K_1+K_2=8 \\ \dot{x}(0)=-4K_1-8K_2=-44 \end{cases} \quad \begin{cases} K_1=5 \\ K_2=3 \end{cases}$$

$$\therefore x(t)=5e^{-4t}+3e^{-8t}, \quad t\geq 0$$